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DYNAMICS OF DISCHARGING OF A HEAT ACCUMULATOR IN AN INFINITE GROUND MASSIF

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Based on the approaches presented in [Inzh.-Fiz. Zh., **77**, No. 4, 10–19 (2004)], *the problem on discharging of the heat accumulated earlier by single heat exchangers and their combination in an infinite ground massif has been solved. The inefficiency of the accumulation and discharging by single heat exchangers and the high degree of recovery of the accumulated heat in the case of their clustered arrangement have been established.*

Among the promising trends in nontraditional power engineering is the use of solar energy accumulated during a warm season for heating of rooms and hot-water supply during a cold season. The required large capacity of accumulators (storage cells) makes it necessary to focus attention on natural objects, one of which can be a ground massif. Because of the high thermal resistance of the ground, creation of heat-insulated ground accumulators is impractical. In this case, one must solve problems of accumulation and discharging of heat in an infinite ground massif. This feature, with allowance for the discontinuous character of operation of solar collectors, makes it difficult to directly solve the fundamental equation of nonstationary heat conduction even by numerical methods. The arising problems have been analyzed in detail in [1], where a combined integral method of solution of problems of ground accumulation of heat by coaxial and U-shaped heat exchangers vertically arranged in the ground and in which an intermediate heat-transfer agent, i.e., water heated by solar collectors, circulates, has been proposed and realized; the accumulation by single heat exchangers and their combination has been considered. In this work, we present, based on the approaches of [1], the solution of inverse problems: those of recovery of heat from heated ground massifs by heat-exchange systems used in accumulation.

1. Discharging by Single Heat Exchangers. The diagrams of heat exchangers are identical to those presented in [1] (Fig. 1). The order of calculations and the form of functions are the same as in accumulation if the heat-flux density q_0 is interpreted as a vector. According to [1], a single-parameter family of temperature-distribution functions in a ground massif has the form

$$\frac{T - T_{\rm m}}{T_0 - T_{\rm m}} = (1 - \eta)^3 \, (1 + 3\eta - A_{\rm m}\eta) \, , \tag{1}$$

where

$$\eta = \frac{r - R_0}{R - R_0} \, ; \quad A_{\rm m} = \frac{q_0 \, (R - R_0)}{\lambda_{\rm m} \, (T_0 - T_{\rm m})} \, ; \quad q_0 = \alpha_0 \, (T_{\rm wat} - T_{\rm w}) \, ; \quad {\rm Bi}_{\rm m} = \frac{\alpha_0 \, (R - R_0)}{\lambda_{\rm m}} \, . \tag{2}$$

Unlike accumulation [1], the set of $A_{\rm m}$ values in discharging lies in the interval $\{-\infty, 0\}$. Then the distribution (1) must have its maximum on the segment $0 \le \eta \le 1$. Investigation of (1) for extremum leads to the relations

$$\eta_{\rm ex} = \frac{A_{\rm m}}{4A_{\rm m} - 12} \, , \quad T_{\rm ex} = T_{\rm m} + (T_0 - T_{\rm m}) \, (1 - \eta_{\rm ex})^3 \, (1 + 3\eta_{\rm ex} - A_{\rm m}\eta_{\rm ex}) \, , \tag{3}$$

according to which $\eta_{\rm ex}$ is monotonically dependent on $A_{\rm m}$. The radius $R_{\rm ex}$ corresponding to $T = T_{\rm ex}$ is equal to

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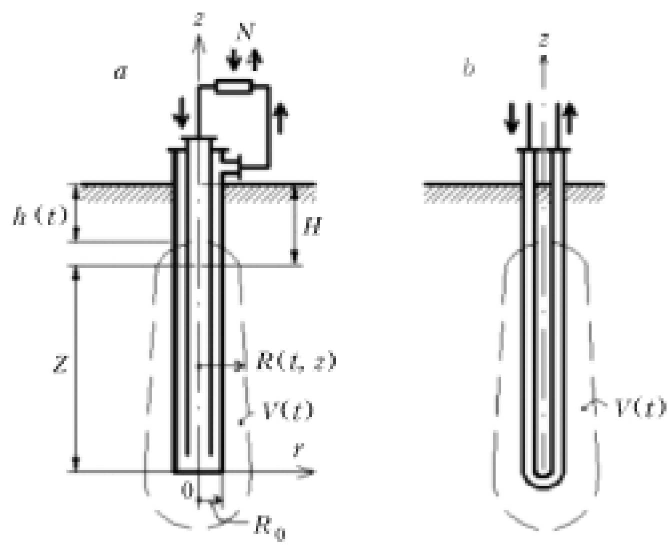


Fig. 1. Diagrams of coaxial (a) and U-shaped (b) heat exchangers.

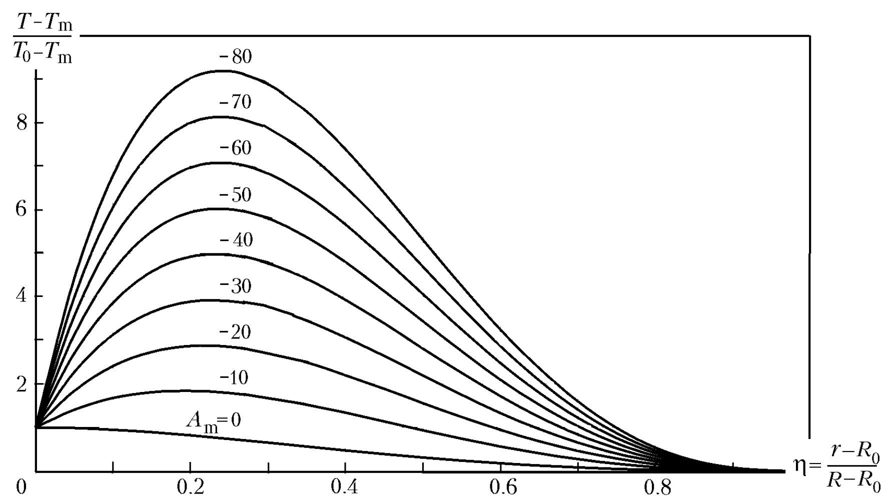


Fig. 2. Temperature distribution in the ground massif in discharging.

$$R_{ex} = R_0 + \eta_{ex}\,(R - R_0)\,, \tag{4}$$

and the domain of definition of η_{ex} is $0 \le \eta_{ex} \le 0.25$. The temperature profiles (1) are presented in Fig. 2. Heat propagates in the positive direction of r when $r > R_{ex}$; when $r < R_{ex}$ we have the inflow of heat to the heat exchanger. Thus, $T_{ex}(R_{ex})$ influences both processes and is the determining quantity.

To solve the problem of discharging with prescribed $T_m(z)$ and $\lambda_m(z)$ we must find

$$T_{wat}\,(t, z)\,,\ \ T_w\,(t, z)\,,\ \ T_0\,(t, z)\,,\ \ T_{ex}\,(t, R_{ex})\,,\ \ R\,(t, z)\,,\ \ R_{ex}\,(t, z)\,,\ \ \alpha_0\,(t, z)\,,\ \ A_m\,(t, z)\,. \tag{5}$$

The functions (5) are determined by

(1) the energy equation of the heat-transfer agent

$$\frac{\partial T_{wat}}{\partial z} = -\,2\pi R_w\,\frac{\alpha_w\,(T_{wat} - T_w)}{G_{wat} c_{wat}}\,; \tag{6}$$

(2) the equation of heat transfer through the heat-exchanger wall

$$q_0 = p_w (T_w - T_0) , \quad p_w = \frac{\lambda_w}{R_0 \ln \left(\frac{R_0}{R_w} \right)} ; \tag{7}$$

(3) the equation of variation in the massif temperature when $r = R_{ex}$

$$\frac{\partial T_{ex}}{\partial t} = 6a_m \frac{T_0 - T_m}{(R - R_0)^2} (1 - \eta_{ex}) [- 2 + 6\eta_{ex} + (1 - 2\eta_{ex}) A_m] ; \tag{8}$$

(4) the equation of conservation of the energy accumulated by the ground

$$\int_0^t dt \int_0^Z 2\pi R_0 q_0 dz = \int_0^Z dz \int_{R_0}^R 2\pi\rho_m c_m (T - T_m) \, rdr + \int_{R_0}^{R(0)} 2\pi\rho_m c_m (T - T_m) \, r^2 dr + \int_{R_0}^{R(Z)} 2\pi\rho_m c_m (T - T_m) \, r^2 dr , \tag{9}$$

where $T - T_m$ is found according to (1), and the heat-flux density q_0 is determined from the third formula of (2);

(5) expression (2) for A_m;

(6) the first equation of (3) and formula (4) for determination of R_{ex};

(7) the second equation of (3) for determination of T_0 at the known temperature T_{ex};

(8) the heat-transfer coefficient α_w on the wall ($r = R_w$) and its equivalent value on the surface

$$\alpha_0 = \alpha_w \frac{R_w}{R_0} , \tag{10}$$

which is determined in simultaneous solution of the thermal and hydrodynamic problems with the use of the existing dependences for heat exchangers of the types in question.

As has been noted in [1], the temperature of the heat-transfer agent is equalized over the height z in coaxial and U-shaped heat exchangers. Consequently, in most practical cases the calculated parameters of the process can be considered to be constant with height, and the difference in the temperatures of water at the inlet $T_{wat.b}$ and outlet $T_{wat.end}$ of the heat exchanger can be determined according to (6). The mean value of these temperatures corresponds to a calculated T_{wat}. As far as the "quality" of the energy produced is concerned, it is characterized by the coefficient of performance of a heat pump. In operation of the pump on the Carnot cycle, we have

$$\varepsilon_{pump} = \frac{T_{wat} + 273}{T_{pump} - T_{wat}} , \tag{11}$$

where T_{pump} is the temperature at the pipe outlet; it is taken to be 55^{o}C here. The discharging efficiency is determined by the relation

$$\eta_t = \frac{Ei_{end}}{Ei_{in}} , \tag{12}$$

where Ei_{in} is the excess initial energy of the massif and Ei_{end} is the final value of the recovered energy.

The initial thermal characteristics of the massif discharged are known from solution of the problem of accumulation. The value of q_0 is prescribed and is usually constant. Discharging is carried out continuously. Therefore, unlike the accumulation problem, there are no difficulties with formulation of the initial and boundary conditions for the problem in question.

The solution of the problem on recovery of the accumulated heat can only be obtained by a numerical method. Subsequently we have performed calculations for the same characteristics of the interacting systems as in [1]:

(a) the coaxial heat exchanger: R_0 = 0.054 m, R_w = 0.050 m, r_w = 0.040 m, λ_w = 17.5 W/(m·K), Z = 50 m, G_{wat} = 5.0 kg/sec, $T_{wat} \sim 50^o$C, Re = $0.67 \cdot 10^5$, $\alpha_0 = 0.86 \cdot 10^5$ W/(m^2·K), v_{wat} = 1.79 m/sec, and ζ = 0.02;

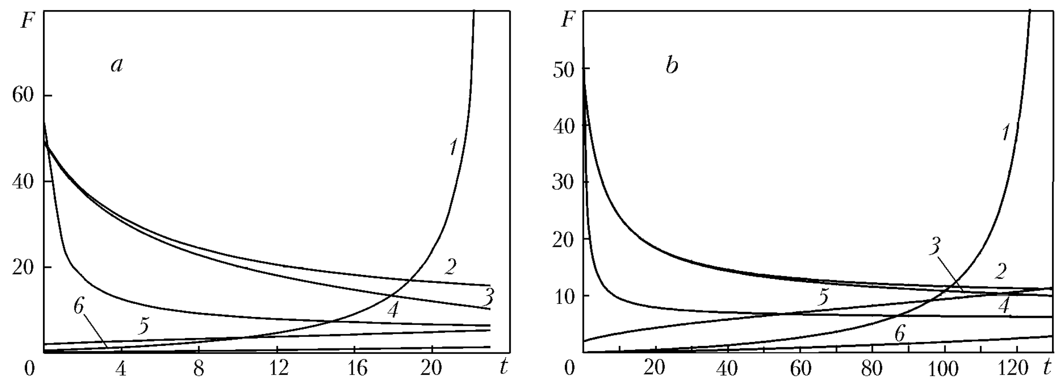


Fig. 3. Change in the discharging parameters F for $q_0 = -10$ W/m^2 (a) and $q_0 = -1$ W/m^2 (b): 1) $-A_m$; 2) T_{ex}, ^oC; 3) T_0, ^oC; 4) ε_{pump}; 5) R, m; 6) R_{ex}, m. t, days.

(b) the ground: $\rho_m = 1.84 \cdot 10^3$ kg/m^3, $\lambda_m = 1.42$ W/(m·K), $c_m = 1.15 \cdot 10^3$ J/(kg·K), and $T_m = 10^oC$.

It is noteworthy that, despite the possible variety of the composition of grounds, their determining thermophysical characteristics change only slightly. Thus, the specific heat capacity per unit volume takes on values in the range $(1.2–2.6) \cdot 10^3$ W/(m^3·K), whereas the thermal diffusivity takes on values in the range $(0.1–0.9) \cdot 10^{-6}$ m^2/sec. Thus, the parameters given above are quite characteristic of grounds.

Below, we give results of calculations of the recovery of heat accumulated earlier in controlled operation of a coaxial heat exchanger with a 24-hour accumulator [1] (for the final parameters $T_0 = 50^oC$ and $R = 2.0$ m). The temperature profile corresponded to the condition $A_m \approx 0$, and the energy characteristic was $ei_{in} = Ei_{in}/Z = 0.2237 \cdot 10^9$ J/m. The recovery of heat stopped if the temperature of water in the heat exchanger decreased to the initial temperature of the ground massif.

Figure 3a shows the dynamics of the process for $q_0 = -10$ W/m^2. The discharging was completed on the 23rd day from the beginning of operation. The final value $T_{ex} = 15.66^oC$ points to the considerable quantity of the excess energy left in the massif. A comparatively low value — $T_{pump} = 55^oC$ — has led to an acceptable "quality" of the energy ($\varepsilon_{pump} > 6.3$). The radius of propagation of heat due to its continuous "drift" attained 5.26 m; the value of η_t turned out to be equal only to 3.06%. Figure 3b gives results of discharging for $q_0 = -1$ W/m^2; the operating time of the heat exchanger increased and was 130 days. Despite the low final value $T_{ex} = 11.20^oC$, the increase in the thermal-massif radius to $R = 11.41$ m made it possible to recover only 1.71% of the energy accumulated. A monotonic change in the final parameters with variation of q_0 from -1 to -10 W/m^2 (Fig. 4) gives no grounds to develop an algorithm of time changes in q_0 that would substantially improve the index η_t. Thus, both the accumulation of energy by a single heat exchanger [1] and its discharging should be considered to be inefficient.

2. Discharging by a Combination of Heat Exchangers. According to [1], at the end of ground accumulation of heat by a cluster of $k = m \times n$ heat exchangers, the potential T_{bas} of the basic accumulation region of volume V_{bas} approaches a maximum possible value, whereas in the buffer subregion V_s adjacent to the basic one, the temperature monotonically changes from T_{bas} to T_m on the width R_s. In discharging of a thermal massif, peripheral heat exchangers are not operated with the aim of maintaining a uniform temperature distribution in V_{bas} and the number of heat exchangers operating with the same thermal load (q_0 = idem) is found as

$$k_{op} = mn - 2(m + n) + 4 . \tag{13}$$

In what follows, we assume that q_0 is a positive value determined by its modulus.

During the first one to two hours of operation of the cluster, we will have the contact of the radii of thermal action of the heat exchangers, just as in accumulation; from this instant, the heat will be recovered from V_{bas} when $R = R_j$ = const (Fig. 5). The temperature profiles are similar to those in accumulation [1]:

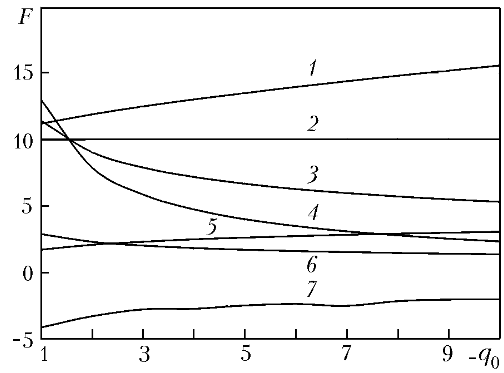


Fig. 4. Change in the final parameters of discharging F: 1) T_{ex}, °C; 2) T_0, °C; 3) R, m; 4) duration, decade; 5) η_t, %; 6) R_{ex}, m; 7) $-A_m \cdot 10^{-3}$. q_0, W/m^2.

$$\frac{T_j - T}{T_j - T_0} = \begin{cases} (1-\kappa)^2 (1 + 2\kappa - A_j \kappa) , & 0 \le A_j \le 3 ; \\ (1-\kappa)^{A_j} , & 3 < A_j \le \infty , \end{cases} \tag{14}$$

where

$$\kappa = \frac{r - R_0}{R_j - R_0} ; \quad A_j = \frac{q_0 (R_j - R_0)}{\lambda_m (T_j - T_0)} , \tag{15}$$

and the rate of change in T_0 is calculated as

$$\frac{dT_0}{dt} = a_m \frac{(T_j - T_0)}{(R_j - R_0)} \left[\frac{6 - 4A_j}{(R_j - R_0)} + \frac{A_j}{R_0} \right] , \quad 0 \le A_j \le 3 ;$$
$$\frac{dT_0}{dt} = a_m \frac{(T_j - T_0)}{(R_j - R_0)} \left[\frac{A_j (1 - A_j)}{(R_j - R_0)} + \frac{A_j}{R_0} \right] , \quad 3 < A_j \le \infty . \tag{16}$$

In the buffer subregion, the dynamics of the processes is no different from their character in accumulation. The temperature distribution and the rate of change in the parameter R_s have the form [1]

$$\frac{T - T_m}{T_j - T_m} = (1 - \psi)^3 (1 + 3\psi) , \quad \psi = \frac{u}{R_s} , \quad u = x, y ;$$
$$\frac{dR_s}{dt} = \frac{12 a_m}{R_s} . \tag{17}$$

Here u is counted off from the surface along the external normal to S_{bas} relative to the volume V_{bas}. The density of the heat flux absorbed by the buffer subregion is determined by the expression

$$q_s = \frac{d}{dt} \int_0^{R_s} \rho_m c_m (T - T_m) \, du = 0.4 \rho_m c_m \left[(T_j - T_m) \frac{dR_s}{dt} + R_s \frac{dT_j}{dt} \right] . \tag{18}$$

When the value of (18) is negative, the heat flux is directed from the buffer subregion to the basic region. The heat balance of the basic region of the thermal massif is as follows:

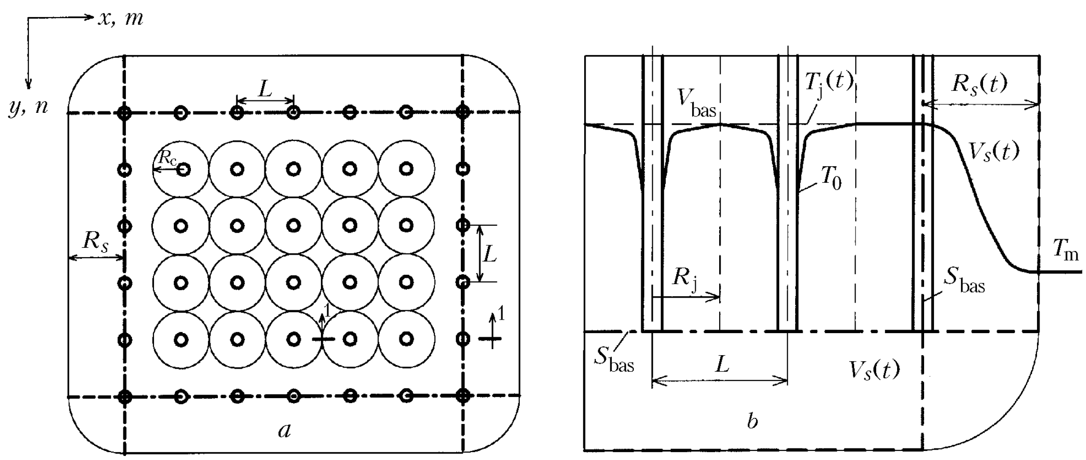


Fig. 5. View of the cluster of heat exchangers (a) and cross section along 1 – 1 at $t > t_j$ with the temperature distribution in the massif (b).

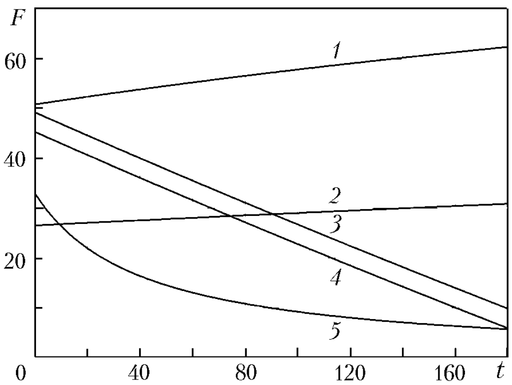
$$\frac{d}{dt}\left[\rho_m c_m (T_j - T_m) V_{bas} - k_{op} Z \int_{R_0}^{R_j} 2\pi\rho_m c_m (T_j - T)\, r dr\right] = - S_{bas} q_s - 2\pi k_{op} R_0 Z q_0 . \tag{19}$$

Having the distribution (14), we can express the integral in (19) in quadratures. The system of equations (14)–(19) is closed and makes it possible to find $T_j(t)$, $T_0(t)$, $R_s(t)$, $q_s(t)$, and other parameters of the process, which are determined by these functions, for the prescribed q_0 and the known initial conditions and data on the heat exchangers.

Let us select the case approaching that of practical importance as a computational example. Since the initial thermal efficiency of a cluster [1] $\eta_{t.cl} = 1 - (m + n - 1)/mn$ increases with increase in the number of heat exchangers $k = m \times n$, it is most efficient to use ground accumulation of heat for heating and hot-water supply of a village with several thousand people. In such a village, there is usually a school or village stadium an area of whose football field of the order of 100 × 100 m can be used for underground location of heat exchangers. Ordinary solar collectors make it possible to obtain an operating-water temperature no higher than 60°C. Therefore, the maximum attainable temperature in the basic region V_{bas} will be of the order of $T_{j,max} \sim 50$°C in accumulation. The highest potential of heat must be strived for in its recovery. If we allow for the considerable thermal resistance of a typical ground massif whose characteristic has been given above in Sec. 1, we should be oriented, in accumulation, to a heat-flux density of the order of 300–400 W/m^2 at the beginning of the process with a step-by-step reduction ensuring the final potential of the massif $T_j \approx T_{bas} \approx 50$°C and to $q_0 \approx$ 50–80 W/m^2 in discharging. Since the limiting link of heat transfer is the resistance of the ground, it is impractical to maintain high α_0 values ($\approx 10^5$ W/m^2) and the transition from the viscous-gravitational regime of flow of the intermediate heat-transfer agent to a turbulent regime is necessary. The positive aspect of the latter decision lies in decreasing considerably the water flow rate in the circuit.

With allowance for what has been presented above, we have the following characteristics of a coaxial heat exchanger: R_0 = 0.110 m, R_w = 0.100 m, r_w = 0.074 m, λ_w = 17.5 W/(m·K), Z = 100 m, G_{wat} = 0.33 kg/sec, Re = 1664, α_0 = 198.2 W/(m^2·K), v_{wat} = 0.023 m/sec, and ζ = 0.058. The parameters of a cluster of heat exchangers on a 100 × 100 m site are as follows: $m = n = 51$, L = 2.0 m, and $\eta_{t.cl}$ = 0.961. The thermophysical properties of the ground are analogous to those given in Sec. 1. The accumulation of heat for q_0 = (380–20·dec) W/m^2 (here dec = $\{1, 2, ..., 18\}$ is the decade No.) led to a change in the temperature T_{wat} of the heat-transfer agent from 37.07°C (at the beginning of the first decade) to 50.87°C (at the end of the eighteenth decade) at a maximum value of 56.99°C falling within the tenth decade. The final indices are as follows: the linear parameter R_s = 26.55 m, the temperature T_j = 49.24°C, and the quantity of accumulated energy $Ei = 0.1361 \cdot 10^{15}$ J.

The dynamics of continuous discharging for q_0 = 53 W/m^2 = const is presented in Fig. 6. The difference $T_j - T_{wat}$ was no higher than 4°C; the temperature of the heat-transfer agent decreased from 45.32 to 5.88°C. The parameter R_s increased by 4 m in discharging and attained a value of 30.91 m. The "quality" of the energy produced,



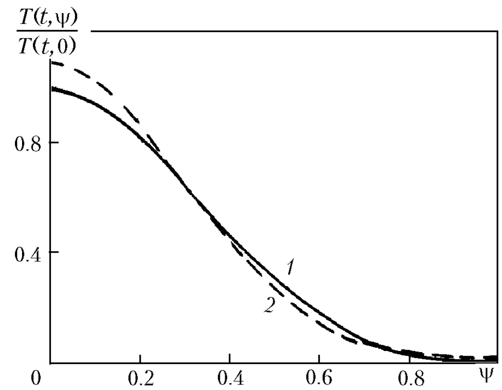


Fig. 6. Change in the discharging parameters F by a cluster of heat exchangers: 1) $-q_s$, W/m^2; 2) R_s, m; 3) T_j, ^{o}C; 4) T_{wat}, ^{o}C; 5) ε_{pump}. t, days.

Fig. 7. Temperature distribution in pulsed heating of the bar: 1) according to the proposed procedure; 2) according to the classical solution $T(t, 0)$ has been determined from (23).

which is characterized by the coefficient of performance of ε of the heat pump, turned out to be high. Noteworthy is a 100% recovery of the energy accumulated.

Based on the heating rate of 60 W/m^2 per unit living floor area (intrafloor heating system), a living density of 20 m^2/man, and an energy consumption by hot-water supply of 250 W/man, accumulated (and then recovered) solar energy in the quantity $Ei = 0.1361 \cdot 10^{15}$ would suffice to meet the needs of a village with a population of 6000 people for 180 days.

The data given here are, unambiguously, evidence in favor of the cluster-type solution of accumulation and discharging, not in favor of "single" operation of thermal devices.

3. Comparison of the Methods of Solution. Let us compare the method proposed here and in [1] for solution of problems of nonstationary heat conduction to classical methods of solution. A comparison for spatially bounded bodies has been given in [2] and points to the equivalence of both approaches in mathematical terms. We analyze them for the cases of infinitely extended objects. Naturally, we cannot expect a complete quantitative coincidence of the results, as in [2]. The reason is mainly the finite radius $R(t)$ of propagation of heat, which has been introduced here, just as in [1]; this seems more correct from the physical viewpoint than $R \to \infty$ but comes into conflict with the properties of the fundamental heat-conduction equation. For this reason, the fullness of the temperature profile for a finite $R(t)$ must be larger than that for $R \to \infty$ on condition that the energy balance is strictly observed. Below, we give such a comparison using, as an example, the problem on pulse heating in the ε vicinity of the cross section with $x_0 = 0$ (on the portion $-\varepsilon < x_0 < +\varepsilon$) of an infinite bar with a constant cross section and lateral heat insulation. The classical solution is known to have the form

$$T(t, x) = \frac{2\varepsilon T(0, 0)}{2\sqrt{\pi a_m t}} \exp\left(-\frac{(x-\xi)^2}{4a_m t}\right), \tag{20}$$

where $(x_0 - \varepsilon) < \xi < (x_0 + \varepsilon)$ and $-\infty < x < +\infty$. According to the procedure proposed, we have

$$T(t, x) = \frac{2\varepsilon T(0, 0)}{0.8 R_s(t)} (1-\psi)^3 (1+3\psi), \tag{21}$$

where $\psi = x/R_s$, $(x_0 - R_s) \le x \le (x_0 + R_s)$ and, according to (17), $R_s = \sqrt{24 a_m t}$. We take $\varepsilon \to 0$, $T(0, 0) \to \infty$, and $2\varepsilon T(0, 0) = 1$, as is customary in classical solutions. Let us transform (20), separating the group corresponding to the expression for $R_s(t)$ in it, and obtain an expression equivalent to the classical solution

$$T(t, \psi) = \frac{1}{0.724 R_s} \exp(-6\psi^2), \quad -\infty < \psi < +\infty . \tag{22}$$

Then formula (21) will take the form

$$T(t, \psi) = \frac{1}{0.8 R_s} (1 - \psi)^3 (1 + 3\psi), \quad 0 \le \psi \le 1 . \tag{23}$$

We note, first of all, the structural similarity of both dependences. The preexponential factor in (22) and the coefficient corresponding to it in (23) yield a value of $T(t, 0)$ that is 0.8/0.724 = 1.106 times higher in classical solution than that in solution according to the procedure proposed. A comparison of (22) and (23) on the segment $0 \le \psi \le 1$ is given in Fig. 7 and is in complete agreement with the above prediction.

Thus, modeling (performed here and in [1]) of the dynamics of ground accumulation and discharging of heat has revealed the high operating efficiency of a combination of heat exchangers as compared to single systems.

NOTATION

A, parameter; a, thermal diffusivity, m^2/sec; c, specific heat, J/(kg·K); Ei, energy, J; G, flow rate of the intermediate heat-transfer agent, kg/sec; H, height of the heat-insulated portion of a heat exchanger (Fig. 1), m; h, height of the protective ground layer (Fig. 1), m; k, number of heat exchangers in a cluster; L, step, m; m and n, number of heat exchangers in rows parallel to the x and y axes; N, power of the external source (sink), W; q, heat-flux density, W/m^2; R, radius (linear dimension) of propagation of heat, m; Re, Reynolds number; S, area of the heat-transfer surface, m^2; T, temperature, ^{o}C and K; t, time, sec; V, volume, m^3; v, velocity, m/sec; x, y, z and r, z, u, coordinates, m; Z, operating height of the heat exchanger, m; α, heat-transfer coefficient, W/(m^2·K); ε_{pump}, coefficient of performance of the heat pump; ε, infinitesimal, m; λ, thermal conductivity, W/(m·K); ξ, coordinate, m; η and ψ, dimensionless coordinates; η_t, thermal-efficiency coefficient; ζ, coefficient of hydraulic resistance; ρ, density, kg/m^3. Subscripts: 0, parameters for $r = R_0$; water, water; in, initial; e, end; m, massif; b, beginning; bas, basic; op, operating, j, parameters in joint operation of heat exchangers, t, thermal (heat); w, wall, parameters of the interior heat-exchanger wall; cl, cluster of k exchangers; ex, extremum; s, quantities counted off from S_{bas}; pump, pump; max, maximum.

REFERENCES

1. A. I. Nakorchevskii, Dynamics of ground heat storage and choice of rational solutions, *Inzh.-Fiz. Zh.*, **77**, No. 4, 10–19 (2004).
2. A. I. Nakorchevskii, Conjugate problems of unsteady heat and mass conduction under varying external conditions, *Inzh.-Fiz. Zh.*, **72**, No. 4, 782–791 (1999).